Math 131B-1: Homework 5

Due: February 10, 2014

1. Read Apostol Sections 4.19-20, 9.1-8.

2. Do problems 4.52, 4.54, 9.2, 9.3, 9.16, 9.21, 9.22 in Apostol. [Note that you may wish to wait until after Monday's lecture to do 9.16.]

3. *Dini's Theorem* Let $f_n : X \to \mathbb{R}$ be a sequence of functions on a compact metric space X which converges pointwise to a continuous function $f : X \to \mathbb{R}$ and suppose that for each x the sequence $\{f_n(x)\}$ is increasing, i.e. $f_n(x) \leq f_m(x)$ for all $n < m$. We will prove that $\{f_n\}$ in fact converges to f uniformly.

   - Let $\epsilon > 0$. For each $n \in \mathbb{N}$, let $g = f - f_n(x)$ and show that $\{x \in X : |g(x)| < \epsilon\}$ is an open set V_n^ϵ of X. Moreover, show that for $n < m$, we have the inclusion $V_n^\epsilon \subseteq V_m^\epsilon$.
   - Show that $X \subset V_N^\epsilon$ for some $N > 0$. [Hint: the V_N^ϵ cover X.]
   - Prove that $f_n \to f$ uniformly.
   - Give an example showing that the theorem is not true if we do not require f to be continuous. [Hint: Consider the examples done in lecture.]

   This is one of very few situations where pointwise convergence implies uniform convergence.